

TWO-DIMENSIONAL MOTION OF VAPORS WHEN
ENERGY IS SUPPLIED TO THE OUTER LAYERS
OF THE VAPORIZED SUBSTANCE AND WHEN
THERE IS RADIATIVE ENERGY TRANSFER TO
THE SUBLIMATING SURFACE

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Self-similar motions of vapors due to laser radiation were discussed in [1-3]; self-similar conditions for transparent vapors [1], self-consistent conditions of motion and heating of a vaporized substance [2], in which the optical thickness of the vapors is of the order of unity, and also the self-similar motions of a heat conducting gas heated by laser radiation [3].

Below we discuss the two-dimensional homogeneous motion of vapors of a cold, absolutely rigid body when energy is supplied to the outer layers of the vaporized substance and there is radiative energy transfer to the sublimating surface.

The system of equations in gas dynamics with energy input and radiative transport of the vapors can be written in the form of integral conservation equations

$$\oint V dm + u dt = 0 \quad \oint u dm - p dt = 0 \quad \oint (\varepsilon + \frac{1}{2} u^2) dm - (pu + q) dt = - \int_s f dm dt \quad (1)$$

Here p is the pressure, V is the specific volume, u is the gas velocity, ε is the internal energy of unit mass, f is the intensity of heat input to the gas per unit mass and unit time, q is the radiative energy flux, m is the mass (Lagrangian) coordinate, and t is the time; the integral on the right side is taken over an area bounded by an arbitrarily chosen contour.

The equation of state of the vapors is assumed to have the form

$$\varepsilon = \frac{p}{\rho (\gamma_{\text{eff}} - 1)} \quad (2)$$

where ρ is the gas density and γ_{eff} is the effective adiabatic index.

The radiative energy flux q in the vapors is computed from the equation [4]

$$q = F^+ - F^-, \quad F^+(\tau) = \exp(-\frac{3}{2}\tau) \int_0^{\tau} \sigma T^4 \exp(\frac{3}{2}t) \frac{3}{2} dt$$

$$F^-(\tau) = \exp(\frac{3}{2}\tau) \int_{\tau}^{\infty} \sigma T^4 \exp(-\frac{3}{2}t) \frac{3}{2} dt, \quad \tau = \int_0^m \frac{dm}{\rho l} \quad (3)$$

Here F^+ and F^- are the radiative energy fluxes in the positive and negative directions, respectively; T is the gas temperature; τ is the optical thickness of the vapors; l is the Rosseland radiation range; and σ is the Stefan-Boltzmann constant.

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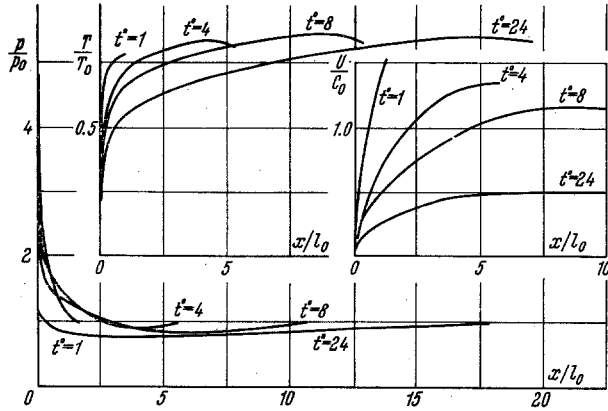


Fig. 1

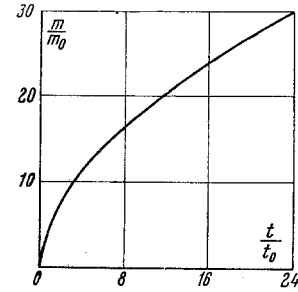


Fig. 2

The motion of the sublimation boundary m_1 is governed by the equation

$$\frac{dm_1}{dt} = \frac{q(m_1)}{Q}, \quad m_1(0) = 0 \quad (4)$$

where Q is the sublimation energy.

If during dispersion due to radiative energy transport the temperature and velocity of the vapors significantly exceed the temperature and velocity at the sublimation boundary, while the density is much less than that at the sublimation boundary, when $m = m_1$ we can assume the following boundary conditions:

$$u_1 = 0, \quad T_1 = 0, \quad \rho_1 = \infty \quad (5)$$

The pressure p_1 at the sublimation boundary and the mass flux $dm_1/dt = \rho_1 u_1$ are finite and are determined during the solution.

At the dispersion boundary ($m = 0$) the following condition is assumed:

$$p(0, t) = p_0 = \text{const} \quad (6)$$

It is also assumed that the energy is liberated at the mass layer m_* and reaches the dispersion boundary

$$\int_0^{m_*} f(m, t) dm = q_0, \quad f(m, t) = 0 \quad \text{when } m > m_* \quad (7)$$

In computing the mass numerically, m_* was made equal to the first step length in the mass coordinate which made it possible to leave the relation between the energy input $f(m, t)$ and the layer m_* undefined.

The system of equations (1) with boundary conditions (4) and (5) was solved by the method proposed by Godunov for solving nonstationary adiabatic problems in gas dynamics [5] and generalized by Fonarev [6] to the case of nonadiabatic motions.

We give the computation of the vapor parameters from an iron plate. The thermodynamical characteristics of the vapors $\varepsilon(\rho, T)$, $\gamma_{\text{eff}}(\rho, T)$, and the Rosseland radiation range $l(\rho, T)$ were computed on the basis of [7, 8].

The relation between the dimensionless variables (pressure $P = p/p_0$, temperature $T^\circ = T/T_0$, and velocity $U = u/c_0$) and the dimensionless coordinates $X = x/l_0$ for a number of moments of time $t^0 = t/t_0$ are shown in Fig. 1. The typical parameters are pressure at the dispersion boundary p_0 ; temperature T_0 defined by the flux of the energy input q_0 ; the speed of sound c_0 ; the typical length is the radiation range l_0 ; the typical time is t_0 .

Figure 2 shows the relation between the nondimensional time and the reduced sublimated mass.

The results of the numerical calculations for the problem proposed showed that intense input of energy into the outer layers of a vaporized substance is accompanied by a significant heating of the outer layers which leads to radiative energy transport to the inner layers and the subliming surface.

Two stages can be detected in the motion of the vaporized substance. In the acceleration stage the pressure at the subliming surface p_1 can be considerably greater than that at the dispersion boundary p_0 ($t^0 = 1$).

In the braking stage ($t^0 = 4, 8, 24$) the velocity of the vapors falls and the pressure in the vapors tends to the pressure p_0 . The optical thickness of the vapors increases with time and considerably exceeds unity. The temperature distribution in the vapors (Fig. 1) has a large plateau, falling sharply at the sublimating surface. The mass temperature $m_*(T^*)$ can be much greater than the temperature of the fundamental mass of gas.

Radiation of the vaporized substance in vacuum increases with time and is a considerable part of the energy input to the outer layer. For sufficiently large values of the time ($t^0 = 24$) radiation of the vaporized substance in vacuum tends to the value of the energy input.

The above calculation showed that in the conditions under discussion the radiative energy flux to the sublimating surface is much less than the energy flux liberated at the outer layers of the vaporized substance, i.e., the sublimating surface is strongly screened by the material vapors. As a result of this the parameters at the sublimating surface, and also the parameters of the fundamental mass of the vaporized substance, do not depend strongly on the way in which energy is liberated in the outer layers (as a function of the mass coordinate).

We compare the conditions of motion of the vapors discussed above with the self-consistent conditions of [2] which are also nonstationary. In the case of self-consistent conditions the radiation flux at the sublimating surface is independent of the time (when the external source is of constant power) and is approximately 0.75 of the flux from the external source, which corresponds to an optical thickness for the vapors of $\tau \approx 0.28$.

In the conditions discussed the optical thickness of the vapors was much greater than unity and the flux to the sublimating surface was much less than the energy liberated in the outer layers.

The pressures at the sublimating surface (the pressure p_1 is several times greater than that at the dispersion boundary p_0) are not markedly different (not more than by a factor of two), but the vaporized mass in the case of self-consistent conditions is much less (by an order of magnitude) than that in the case of energy input to the outer layers. This is explained by the fact that in the case of self-consistent conditions the temperature of the vaporized substance increases with time $T \sim t^{1/4}$ and can considerably exceed that of the vaporized substance in the conditions discussed (the temperature of the fundamental mass of the vaporized substance does not exceed T_0).

A feature of the conditions we have discussed is, as remarked above, the considerable emission in vacuum.

Thus, the proposed conditions of motion for the vapors of a sublimating surface in which energy is supplied to the outer layers of the vaporized substance and is transported to the sublimating surface by radiative transport differs qualitatively from the conditions of motion for vapors considered by previous authors [1-3].

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